Structural Identification of Interval Models of the Static Systems

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Abstract. *There is the method of structural identification of "input-output" model of static system with interval data described in the paper. This method is based on genetic algorithm of synthesis of model structure and on method of ranking generated structural elements, which allows reducing computing complexity of the developed algorithm.*

Keywords

Static system, interval model, structural identification.

1 Introduction

Among mathematical models a special place is occupied by models „input-output", which describe dependence between an input and output variables. For the construction of the indicated class of models the stochastic methods are mainly used, assuming that errors in data, on the basis of which models are built are casual. Lately an interval analysis for the construction of models „input-output" has been often used. In particular, it relates to the cases when statistical descriptions of errors in data are unknown, and only their boundary values are known, when expert data or the imitation modeling data with round errors are applied for the construction of models. Hereby one has to solve two types of tasks: a structural identification of model task as well as a simpler parameter task. Within the limits of stochastic approach to solving structural identification tasks the most essential results has been achieved in the case of using the method of a group consideration of arguments and various algorithms of its realization [1]. However, these methods are useless, when the data are presented as sets of their values, or numerical intervals – interval data.

In this work an approach to solving tasks of structural identification in case of interval data, the method and algorithm of structural identification of such models have been given as well as its application to the tasks of modeling in ecology.

2 The features of application of interval analysis to construction of "input-output" models

Let's consider the basic assumptions on which the methods of interval data analysis are based in the case of construction of models "input-output" of the static systems.

N1. The static system (object) is described by linear parametrical equations

$$y_o = \beta_1 \cdot \varphi_1(\vec{x}) + \ldots + \beta_m \cdot \varphi_m(\vec{x}), \quad (1)$$

where y_o - is a true unknown value of output of the system; $\vec{x} \in R^n$ - is a vector of input variables; $\vec{\beta} = (\beta_1,...,\beta_m)^T$ - is a vector of unknown parameters; $\vec{\varphi}^T(\vec{x}) = (\varphi_1(\vec{x}),..., \varphi_m(\vec{x}))^T$ - is a vector of the known base functions.

N2. The results of the experiment are presented as a matrix X of values of input variables and the proper interval values of an output variable y :

$$X = \{x_{i1},..., x_{in}\}, Y = [y_i^-; y_i^+], i = 1,..., N. \tag{2}$$

They assume that in arbitrary *i*-th supervision the true value of output $y_{oi} = \vec{\varphi}^T(\vec{x}_i) \cdot \vec{\beta}$ belongs to the interval $[y_i^-; y_i^+]$, that is $y_i^- \le y_{oi} \le y_i^+$.

A task of the interval data analysis is the estimation of the unknown vector $\vec{\beta}$, so that the values of the function $y = \vec{\varphi}^T(\vec{x}) \cdot \vec{\beta}$ in points of the experiment belonged to the proper intervals of output. In accordance with the formulated hypotheses, the sought after vector $\vec{b}$ should satisfy such system N of inequalities with m unknown

$$\begin{cases} y_1^- \le b_1\varphi_1(\vec{x}_1) + \ldots + b_m\varphi_m(\vec{x}_1) \le y_1^+; \\ \vdots \\ y_i^- \le b_1\varphi_1(\vec{x}_i) + \ldots + b_m\varphi_m(\vec{x}_i) \le y_i^+; \\ \vdots \\ y_N^- \le b_1\varphi_1(\vec{x}_N) + \ldots + b_m\varphi_m(\vec{x}_N) \le y_N^+; \end{cases} \tag{3}$$

The system (3) is the system N of linear inequalities in relation to m unknown $b_1,\ldots,b_m$. Let the system (3) be compatible. We'll indicate through Ω the set of its solutions, that is

$$\Omega = \left\{\vec{b} \in R^m \middle| \ \vec{Y}^- \le F \cdot \vec{b} \le \vec{Y}^+ \right\} \tag{4}$$

where $\vec{Y}^- = \left\{y_i^-, i = 1,\ldots,N\right\}$, $\vec{Y}^+ = \left\{y_i^+, i = 1,\ldots,N\right\}$ are vectors, made up from the upper and lower limits of intervals $[y_i^-; y_i^+]$, accordingly; $F = \left\{\hat{o}_{ij}, i = \overline{1,N}, j = \overline{1,m}\right\}$ – the known matrix of values of base functions.

The set of decisions Ω generates the set of equivalent (from the point of view of the given interval uncertainty) interval models $\hat{y}(\vec{x}) = \vec{\varphi}^T(\vec{x}) \cdot \vec{b}$, each of which complies with the problem specifications. Hereby, all of interval models are in the corridor:

$$[\hat{y}(x)] = [\hat{y}^-(x); \hat{y}^+(x)], \tag{5}$$

\where $\hat{y}^-(\vec{x}) = \min\limits_{\vec{b}\in\Omega}\left(\vec{\varphi}^T(\vec{x}) \cdot \vec{b}\right)$ and $\hat{y}^+(\vec{x}) = \max\limits_{\vec{b}\in\Omega}\left(\vec{\varphi}^T(\vec{x}) \cdot \vec{b}\right)$ are lower and overhead limits of the functional corridor.

3 The characteristics of preciseness, adequacy, complication and completeness of interval models.

Preciseness of interval model is one of its basic characteristics. The estimation of preciseness requires certain calculation charges. Let's consider preciseness of forecast of a model in a point that is at the fixed set of outputs $\vec{x}$.

Under the forecast of an interval model we'll understand the calculation of output of the system $\hat{y}(\vec{x})$ at the given set of outputs $\hat{y}(\vec{x})$, out of experimental points on the basis of which a model was built, but within the limits of experimental area χ. The basic characteristic of preciseness of an interval model is an error of forecast, which is given as the difference of corridor boundaries (5):

$$\Delta_{y(\vec{x})} = \max_{\vec{b}\in\Omega}\left(\vec{\varphi}^T(\vec{x})\cdot\vec{b}\right) - \min_{\vec{b}\in\Omega}\left(\vec{\varphi}^T(\vec{x})\cdot\vec{b}\right). \quad (6)$$

One can see from the expression (6) that the value of error of forecast depends on the sizes of set Ω. In particular, value $\Delta_{y(\vec{x})}$ in the given point $\vec{x}$ is that less as the distance between the tops of set Ω. If $\vec{b}_P = \vec{b}_s$ for all $p \neq s$, that is a set Ω compresses to the point, the value of error $\Delta_{y(\vec{x})}$ for all of points $\vec{x}$ equals a zero. Taking into account these points of view, preciseness of interval model can be estimated from the point of view of estimation of set Ω dimensions.

In an interval analysis the approximate estimation of set Ω one can get as a rectangular parallelepiped, the verges of which are parallel to the axes of co-ordinates. In this case the got estimations are written down as: $[b_j^-, b_j^+]$, $j = 1,...,m$, where b_j^-, b_j^+ – lower and upper guaranteed limits of possible values of parameters.

Therefore for the estimation of preciseness of models-applicants in the task of structural identification it is to the point to apply the volume of localization hyper-parallelepiped which is described round the set of parameters Ω.

Adequacy of the built models in an interval analysis is provided by compatibility of system (3). Important characteristics are complication of a model which requires quantitative interpretation in the tasks of structural identification.

The existing system of criteria of a structure optimum of interval models is shown in a table 1.

Tab 1. Criteria of optimum of interval models of the static systems

Criteria of complication**:**	
minimization of amount of input variables	$n \to \min$
minimization of amount of model parameters	$m \to \min$
minimization of degree of polynomial	$p \to \min$
Criterion of adequacy:	
compatibility of system of inequalities	$\Omega \neq \varnothing$
Criterion of preciseness:	
minimization of volume *of Π^+*	$V(\Pi^+) \to \min$

As an addition to this system is the given criterion of completeness in the work.

The choice of major managing factors of the system for the reflection of them as a complete set of input variables of an interval model is done on the basis of a completeness criterion.

Let's indicate set of managing factors of the system, from which the sets $\vec{x}_i = (x_{1i}, x_{2i},..., x_{ni})$ have been formed as $\chi_y = \{x_1, x_2,..., x_n\}$, and their sub-sets are reflected in the input variables of models-applicants as $\{\chi_1, \chi_2,..., \chi_l,..., \chi_n\}$, $\chi_1 \subset \chi_2 \subset ... \subset \chi_l \subset ...\chi_n \subseteq \chi_y$, where χ_l is a sub-set having capacity *l*. For example, $\chi_1 = \{x_3\}, \chi_2 = \{x_1, x_2\}$. Accordingly, a sub-vector formed out of the set $\vec{x}_i$ for the sub-set of input variables χ_l of models-applicants we'll indicate as $\vec{x}_{li}$, $\vec{x}_{li} \subset \vec{x}_i$.

The principle of construction of criteria of completeness is based on the property of reducing of variations of intervals of the repeated supervisions in the case of providing a model completeness, which is taken into account additional input variables in the structure which correspond to the most essential real managing factors of the system.

In the case of a passive experiment, the sets $\vec{x}_i, i = 1,..., N$ should be considered as complete, as under the given conditions there is no possibility of changing the set number χ_y of managing factors and, accordingly, a respond of a system to them. Needless to say, that a model will represent the properties of a system which show up in the results of a passive experiment. Therefore its completeness should be viewed from the point of view of the restoration of the properties of the system at the fixed sets of data.

The set of managing factors χ_y is formed by structural elements which appear on the basis of input factors and their interrelation at the following chart: $x_i^k \cdot x_j^k$, $i, j = 1,...,n, k = 0,...,p$, where *p* is a degree of complication (for the polynomial models is a degree of a polynomial). Sub-sets of the reflection of these managing factors (structural elements) into the sets of structural elements of models-applicants will look like: $\{\chi_1, \chi_2, ..., \chi_m, ..., \chi_t\}$, $\chi_1 \subset \chi_2 \subset ... \subset \chi_m \subset ... \chi_t \subseteq \chi_y$, $t = 1,...n \cdot p + \frac{(n \cdot p)!}{2 \cdot (n \cdot p - 2)!}$. For example: $\chi_2 = \{x_2, x_2 \cdot x_3\}$, $\chi_3 = \{x_1, x_2 \cdot x_3, x_2^2\}$.

In this case, a model incompleteness will show up in a variation of the prognosticated interval by a model $[\hat{y}_i^-, \hat{y}_i^+]$, got for the sub-set of structural elements $\chi_m \subset \chi_y$ in relation to the experimental interval of the system response $[y_i^-, y_i^+]$.

Thus, the modification of criterion of completeness should be conducted in the direction of providing minimization of variation of the forecast intervals, got on the basis of a model, and experimental interval data. Hereby, the models-applicants, generated on the basis of sub-sets of input variables $\chi_l \subset \chi_y$ or sub-sets of structural elements $\chi_m \subset \chi_y$, should be adequate. The latter is possible to achieve by way of the increasing a model complication (for the polynomial model an increase of a polynomial degree *p*).

On the basis of the above mentioned, a characteristic of completeness can be presented as follows:

$$R_P(n, m, \chi_n, \Delta) = \frac{1}{N} \sum_{i=1}^{N} \frac{\min\{y_i^+, \hat{y}_i^+(\vec{x}_{li})\} - \max\{y_i^-, \hat{y}_i^-(\vec{x}_{li})\}}{\Delta_{(\vec{x}_{li})}}. \tag{7}$$

where – $\hat{y}_i^-(\vec{x}_{li}) = \min_{\vec{b}_l \in \Omega_l} \left(\vec{\varphi}^T(\vec{x}_{li}) \cdot \vec{b}_l\right)$; $\hat{y}^+(\vec{x}) = \max_{\vec{b}_l \in \Omega_l} \left(\vec{\varphi}^T(\vec{x}_{li}) \cdot \vec{b}_l\right)$ is a lower and upper value of the forecast interval of output variable on the basis of an adequate model-applicant with the set of input variables χ_l; Ω_l is an area of parameters, found from the solution of the system of interval equations (3)

The index of completeness makes it possible to define the degree of model-applicant approximation to optimum. In addition, the indicated characteristic allows determining the degree of influence of separate factors or separate structural element on an output variable, using interval data of the experiment. In number the degree of influence of factors and structural elements can be defined according to their classes.

For structural elements this procedure consists in verification of value of completeness index for interval models with the elementary structure of the kind:

$$[\hat{y}^-\left(x_i^k \cdot x_j^k\right); \hat{y}^+\left(x_i^k \cdot x_j^k\right)] = [\min_{\vec{b}_{l=1} \in \Omega_{l=1}} \left(b_0 + b_1 \cdot x_i^k \cdot x_j^k\right); \max_{\vec{b}_{l=1} \in \Omega_{l=1}} \left(b_0 + b_1 \cdot x_i^k \cdot x_j^k\right)],$$
$$i, j = 1,...,n, k = 1,...,p$$

where an area $\Omega_{l=1}$ is got on the basis of model by a structure $y\left(x_i^k \cdot x_j^k\right) = b_0 + b_1 \cdot x_i^k \cdot x_j^k$ and experimental data

$$x_{si}^k \cdot x_{sj}^k \to [\frac{(y_s^- + y_s^+)}{2} - \Delta_s; \frac{(y_s^- + y_s^+)}{2} + \Delta_s], \; s = 1,...,N,$$

Δ_s is chosen at the terms of compatibility of the system

$$\frac{(y_s^- + y_s^+)}{2} - \Delta_s \le b_0 + b_1 \cdot x_{si}^k \cdot x_{sj}^k \le \frac{(y_s^- + y_s^+)}{2} + \Delta_s, \; s = 1,...,N.$$

The given characteristic of completeness makes it possible to consider the task of structural identification of interval models of the static systems as the task of multi-criterion optimization on the discrete set of models-applicants.

4 Statements of task of structural identification with interval data.

Let's fix the class of models-applicants as polynomial models of the kind

$$y_0(\vec{x}) = \varphi^T(\vec{x}) \cdot \vec{\beta}$$

where $\varphi^T(\vec{x})$ are base polynomial functions in the following kind: $x_i^k \cdot x_j^k$, $i, j = 1,...,n, k = 0,...,p$.

For identification of interval models-applicants interval data are applied as follows:

$$X = \{x_{i1},..., x_{in,}\}, Y = [y_i^-; y_i^+], i = 1,..., N .$$

As a result every model-applicant is characterized by a set

$$\gamma_k : \{m, n, p, \chi_{nk}, R_k, V(\text{Ï}^{\,+}(\Omega_k))\}$$

where m - is a number of model-applicant parameters; n - is a number of input variables (factors); χ_n - is a set of input variables (factors); p - is a maximal degree of a polynomial; R_k - is an index of model-applicant completeness; $V(\text{Ï}^{\,+}(\Omega_k))$ - is a volume of localization hyper-parallelepiped, described round the set of Ω model-applicant parameters.

It should be stressed that in the case when for a model-applicant a set of parameters is empty, that is $\Omega_k = \varnothing$, then the search of characteristic V (the volume of localization hyper-parallelepiped) has no sense. This case arises on condition of incompatibility of the interval system of linear algebraic equations, which is applied in the task of structural identification for finding an area of Ω parameters. There fore in future for the cases when $\Omega_k \neq \varnothing$, we'll make use of the $V(\text{Ï}^{\,+})$ record.

Taking into account the above said, the task of structural identification of interval models "input-output" of the static systems on the basis of experimental data we'll write down as follows:

- for the case of active experiment

$$R_A \xrightarrow{\gamma_k} \max , \; m \xrightarrow{\gamma_k} \min , \; p \xrightarrow{\gamma_k} \min , \; V(\text{Ï}^{\,+}) \xrightarrow{\gamma_k} \min \qquad (8)$$

on condition of compatibility of the system (3);

- for the case of passive experiment

$$R_P \xrightarrow{\gamma_k} \max , m \xrightarrow{\gamma_k} \min , \; p \xrightarrow{\gamma_k} \min , \; V(\text{Ï}^{\,+}) \xrightarrow{\gamma_k} \min \qquad (9)$$

on condition of compatibility of the system (3).

5 Genetic algorithm of structural identification of interval models is a "input-output" of the static systems.

A task of structural identification of interval models is the multi-criterion task of choice on the discrete set of models-applicants.

Such kind of tasks, as a rule, have a set of solutions, depending on the method of aggregate of criteria or sequence of their consideration. Hereby, the classic methods of solving optimization tasks, for example mathematical and, in particular, discrete programming for this case are useless.[2]

Let's formulate the basic principles by which it is possible to realize the neuro-network structure of genetic algorithms of interval models identification.

1. Ranging according to the index of completeness of structural elements of the kind $b_1 \cdot x_i^p \cdot x_j^p,\ i, j = 1,...,n$ is $b_1 \cdot x_i^p \cdot x_j^p,\ i, j = 1,...,n$ at the set level of complication *of p* on the basis of formula (7). Ranks are the basis for determination of sub-set of the most essential structural elements, which will influence the number of generated models-applicants.

2. Forming of initial set of individuals on the basis of which models-applicants will be generated. The choice of individuals is carried out by the help of a threshold selection of structural elements on the basis of their ranging according to the index of completeness.

3. Generating and selection according to the criterion of completeness of models-applicants. In this case the values of index of completeness will correspond to the ranks of models-applicants for the corresponding models.

4. Estimation of complication and preciseness of adequate models-applicants which passed the selection according to the index of completeness.

Fig.2 shows the neuro-network structure of genetic algorithm of identification of the structure of the static systems interval models.

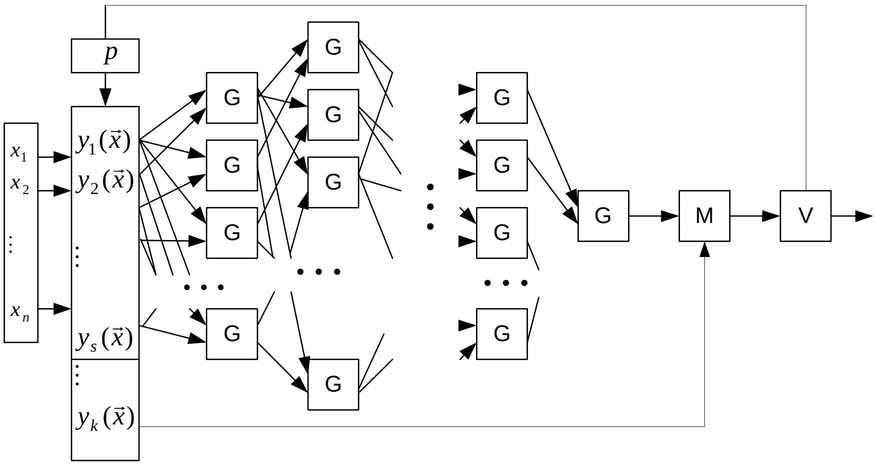

Fig. 2. Neuro-network structure of genetic algorithm with the iteration selection.

where such denotations are used: $x_1, x_2, ..., x_n$ - are input variables, G - is generating and selection of adequate models-applicants according to the index of completeness (adequacy of an applicant model is set by parametrical identification, the structure of a block is given in figure 3); M - is mutation on the basis of replacement (addition) of structural elements of model-applicant by elements which did not pass the selection at the completeness; V - is verification of a model according to complication and preciseness; P - is a management of model complexity.

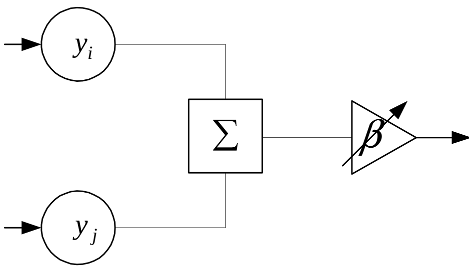

Fig. 3. Structure of a model-applicant generating block.

6 Identifications of a model of the static field of concentrations of harmful emission of oxides of nitrogen.

The tasks related to the ecological monitoring of the environments, vested in the sanitary-epidemiology stations (SES) of cities which have at their disposal measuring apparatus and specially equipped laboratories. One of the main tasks of these laboratories is the control of exceeding permissible rates of atmosphere pollution both by transport and industrial enterprises.

At the found out concentrations of harmful emission in the points of the air measuring it is possible to see the picture of contamination by this substance in the whole town. Hereby from the point of view of the correct estimation of the inflicted losses on the environment an important task is to establish the so-called static fields of concentrations of harmful emission (they can be defined as static as they are changeable to a very little extent during quite a period of time), which form a general picture of morbidity growth in the city as a result of worsening ecological situation.

To make a structural identification of the fields of concentration of harmful emission it is necessary to form the set of input factors which influence the process of measuring concentrations of harmful substances and forming static fields of concentrations of these substances.

Some factors have a high-quality character. That is why it is necessary to raise them to the quantitative values. For this purpose it is possible to apply the method of scales. The factor of the wind force can be not taken into account, as for making the measuring a moderate wind is taken a priori.

The ”measuring point” factor will be expressed via the co-ordinates of points on the map of the city of Ternopil. (fig. 4)

Fig. 4. The fragment of the chart of the city of Ternopil.

Consequently, the set of input variables will contain seven factors $\chi_y = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$, where: x_1, x_2 - are co-ordinates of the point of the air selection for measuring concentration of harmful emission; x_3 - it is the temperature of external environment at the moment of air selection; x_4 - is the air humidity; x_5 - is atmospheric pressure; x_6 - is a direction of the wind; x_7 - is the weather.

Let’s do structural identification of an interval model of the static field of concentration of „oxides of nitrogen”. We’ll search an interval model in the class of polynomial functions.

As a result of passive experiment the retrieval of experimental data was formed as $X \to [\vec{Y}]$, which contains 44 observations (*N=44).* Input variables were rated on the $0 \le x_i \le 1, i = 1, ..., n$

interval. Hereby, the interval initial variable values were got at the following chart: $[y_{0i}^{-} - \Delta_i; y_{0i}^{+} + \Delta_i]$, $\Delta_i = \Delta_{1i}, i = 1,...,N$, where Δ_{1i} is a systematic constituent of error measuring of concentration of „oxides of nitrogen" by devices „Typhoon R-20-2" and „SF-26", the casual constituent of error is excluded, as all factors of influence have been taken into account.

On the fourth stage of generating models-applicants we got the set of adequate models-applicants and making the selection according to the criterion of completeness we carried out the choice of optimum structure of the following kind:

$$y(\vec{x}) = b_0 + b_1 \cdot x_1 + b_2 \cdot x_1 \cdot x_4 + b_3 \cdot x_1 \cdot x_2 + b_4 \cdot x_1 \cdot x_3 + b_5 \cdot x_1 \cdot x_6 + b_6 \cdot x_4 + b_7 \cdot x_6 + \\ + b_8 \cdot x_5^2 + b_9 \cdot x_2 \cdot x_5 + b_{10} \cdot x_1 \cdot x_7 + b_{11} \cdot x_3 + b_{12} \cdot x_5 + b_{13} \cdot x_5 \cdot x_6 + b_{14} \cdot x_2^2$$

where a vector of interval estimations of a model parameters is $\vec{b}$ =([0,2312 0,2403]; [0,5133 0,5358]; [-0,4958 -0,4722]; [-1,2191 -1,1895]; [-0,1518 -0,1378]; [0,6539 0,6857]; [0,1036 0,1217]; [-0,3016 -0,2895]; [-0,3119 -0,3027]; [0,5373 0,5532]; [0,1589 0,1685], [-0,0369 -0,0263]; [0,1007 0,1136]; [-0,6497 -0,6277]; [0,3120 0,3270]).

A forecast interval corridor on the basis of the got model as compared to an experimental corridor is shown in the figure 5.

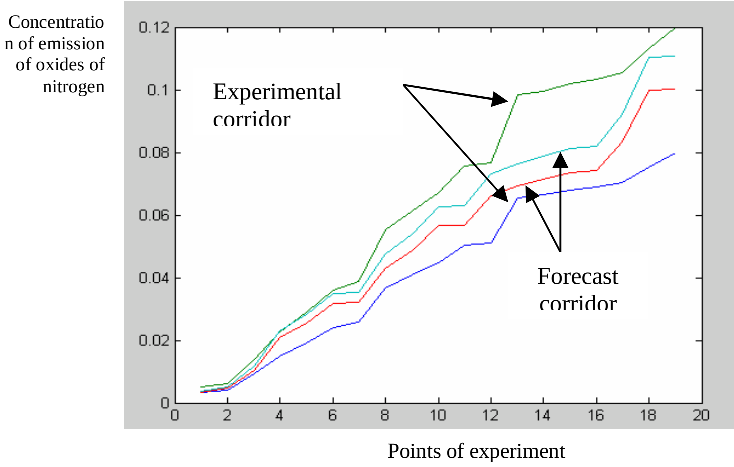

Fig. 5. Corridor of forecast for the year 2006 as compared to experimental.

7 Conclusions

The results of construction of an interval model of the static field of concentrations of oxides of nitrogen have been presented in the work, which is applied in the task of the ecological monitoring for forecast. The got model makes it possible to research the influence of factors of environment on the field of concentration of oxides of nitrogen emission.

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