Image contrast and its connection with fuzzy logic

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Abstract. *The method for determination of image generalized contrast with power character of its elements modification is considered. The expression for determination of image generalized contrast based on power additive generator is ascertained. Theorems about the contrast generator and the rule of contrast addition, based on Yager s-norm, are prooved. For the first time the connection between fuzzy logic and contrast determination through Yager fuzzy conectives is demonstrated. The expression for computation of proposed generalized contrast is searched out. The results of investigations are illustrated by examples.*

Keywords

Image contrast, fuzzy logic, fuzzy connectives, s-norm.

1 Introduction

Image evaluation especially receiving of its quantitative characteristics is one of the actual problems in nowadays. It is caused by the fact that quantitative image evaluation allows to take correct decisions concerning image processing in the automatic mode. While evaluating image quality it should be mentioned that image contrast is one of the principal components of such evaluation. In [1] it was shown that absolute value of contrast as a quantitative value has characteristics of space metric. The rule of contrast addition was determined for the case of contrast linear description [2], that allowed to solve as a whole problem of image generalized contrast definition at linear description of its elements (the case of absolute contrast). Contrast can be represented as quantitative evaluation of reaction of image perception system at its analysis. Therefore it is interesting to find the approaches to determinate image generalized contrast on the basis of its elements contrast with nonlinear description. It would allow to adapt automatic image analysis system to sensor perception characteristics, that realizes analogue-digital transformation of brightness to number, or to carry in correction of contrast sensitivity depending on the region of sensor input value variation and also to construct new algorithms for image quality enhancement by local contrasts amplification.

There are two approaches to quantitative evaluation of image contrast based on absolute [2] and weighted [3] contrasts of its elements. However, generalizations concerning image contrast evaluation for different kinds of its elements contrast are not determined. Therefore the aim of this paper is the determination of generalized approach concerning estimation of image generalized contrast at the certain class of its element contrast, namely for the class of power modification of contrast character with single-valued definition of its maximum value. Assumping this we will firstly consider the contrast of image elements and its determination (Section 2), then determination of generalized contrast of image (Section 3) and its experimental studies (Section 4) and will finish with the Conclusion.

2 Image element contrast and its definition

It is traditionally supposed that contrast is quantitative or qualitative difference between two parts of field of vision, which are visible simultaneously or sequentially. Therefore while evaluating contrast of two image elements as a measure of difference we should demand maximum value of its modulus to correspond to maximum difference of values (gray levels or brightness) L_1 and L_2, and a sign of contrast to indicate only the fact which of the values predominates L_1 or L_2. Moreover modification of contrast absolute value should be symmetric. Analyzing analytical definition of contrast we should mention that its absolute value with the set (space) L of image gray levels should create metric space, since three axioms should be accomplished for it:

1) axiom of equality

$$|C(L_1, L_2)| = 0 \quad \text{if and only if} \quad L_1 = L_2; \tag{1}$$

2) axiom of symmetry

$$|C(L_1, L_2)| = |C(L_2, L_1)|\,; \tag{2}$$

3) axiom of triangle

$$|C(L_1, L_2)| \leq |C(L_1, L_0)| + |C(L_0, L_2)|\,. \tag{3}$$

It is logically to assume that many expressions will satisfy such requirements which practically have to describe the way of distance determination in every specific space.

On the whole, on the one hand we set such requirements to the method of contrast definition of image elements. On the other hand it is known [2, 3], that for the determination of image generalized contrast one should know the way of analytical definition of contrast composition operation for two pairs of elements, i.e. to know the rule of contrast addition. Herewith the usage of different expressions for contrast determination requires stronger requirements than those settled by axiom of triangle (2), namely the equality of contrast sum at two sequential translations $L_1 \to L_0 \to L_2$:

$$C(L_1, L_2) = C(L_1, L_0) \breve{+} C(L_0, L_2), \tag{4}$$

where $\breve{+}$ signifies method of two contrast summation. It means, that for satisfying the condition (4) it is necessary to determine mentioned above rule of contrast addition. For the sake of generality we will assume that fuction of contrast has two arguments defined on the interval [0,1], that is

$$C:\ [0,1] \times [0,1] \to [-1,1]. \tag{5}$$

Reasoning from such definition of two image elment contrasts and necessity of finding the rule of two contrast addition satisfying condition (4), we come to the fact that expression for determination of contrast of two image elements at mentioned above requirements to the characteristics of its maximum value (unique correspondence of maximum contrast absolute value to maximal difference of arguments) is defined by equality:

$$C(x, y) = sign\left(u_a(x) - u_a(y)\right) \cdot \left|u_a^{-1}\left(|u_a(x) - u_a(y)|\right)\right|, \tag{6}$$

where $u_a(\bullet)$ - additive generator of triangular conorm (s-norm) [4], which is the base for construction of fuzzy connectives, $u_a^{-1}(\bullet)$ - function, inverse to additive generator $u_a(\bullet)$. At the same time the absolute contrast, as it was mentioned in Section 1, is often used at image processing. This contrast is based on linear description of two elements difference. With the purpose to generalize approaches to contrast determination it is of interest to use as a basic not only linear functions and but also nonlinear and Equation (6) establishes the relation between contrast and means of fuzzy connectives construction (t-norm and s-norm) as a basis for fuzzy logic formation. Therefore it allows to formulate following theorem.

Theorem 1. The expression based on generator $u_a(x) = x^p$:

$$C(x,y) = sign(x^p - y^p) \cdot \left| |x^p - y^p|^{1/p} \right|, \tag{7}$$

where $p > 0$, is the contrast function and its modulus satisfies conditions (1)-(3).

The proof is evident.

Since the expression for definition of contrast between two image elements with gray levels x and y is determined, it is necessary to find the method of construction of contrast addition rule $\breve{+}$, which should satisfy condition (4). Thereto we will proof following theorem.

Theorem 2. The expression based on s-norm (triangular t-conorm) of Yager

$$\begin{aligned} C(x,z) = sign\left(sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p\right) \times \\ \times \min\left(\left| |sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|^{1/p} \right|, 1\right), \end{aligned} \tag{8}$$

when x, y, $z \in [0,1]$, is analytical expression for addition of two contrasts of appearance (7) by the rule that satisfies requirement (4).

Proof.

To proof it we will rewrite expression (8) in the next form:

$$C(x,z) = \begin{cases} sign\left(sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p\right) \times \\ \quad \times \left| \sqrt[p]{|sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|} \right|, \\ \qquad \text{if } \left| \sqrt[p]{|sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|} \right| \le 1, \\ 1, \qquad \text{if } \left| \sqrt[p]{|sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|} \right| > 1, \end{cases} \tag{9}$$

where

$$C(x,y) = sign\left(x^p - y^p\right) \cdot \left| \sqrt[p]{|x^p - y^p|} \right|, \quad C(y,z) = sign\left(y^p - z^p\right) \cdot \left| \sqrt[p]{|y^p - z^p|} \right|, \tag{10}$$

or

$$\begin{aligned} C(x,z) &= 0.5 \cdot sign\left(sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p\right) \times \\ &\times \left[\left| \sqrt[p]{|sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|} \right| + 1 - \right. \\ &- \left. \left| \left| \sqrt[p]{|sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|} \right| - 1 \right| \right]. \end{aligned} \tag{11}$$

Let us examine the first part of expression (9), namely

$$\begin{aligned} C(x,z) &= sign\left(sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p\right) \times \\ &\times \left| \sqrt[p]{|sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|} \right|, \\ &\qquad \text{if } \left| \sqrt[p]{|sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|} \right| \le 1. \end{aligned} \tag{12}$$

Substituting expressions (10) into (12) we will have

$$
\begin{aligned}
C(x,z) &= sign\left(sign\left(x^p-y^p\right)\cdot\left|\sqrt[p]{|x^p-y^p|}\right|^p + sign\left(y^p-z^p\right)\cdot\left|\sqrt[p]{|y^p-z^p|}\right|^p\right)\times \\
&\times \left|\sqrt[p]{\left|sign\left(x^p-y^p\right)\cdot\left|\sqrt[p]{|x^p-y^p|}\right|^p + sign\left(y^p-z^p\right)\cdot\left|\sqrt[p]{|y^p-z^p|}\right|^p\right|}\right| = \\
&= sign\left(x^p-z^p\right)\cdot\left|\sqrt[p]{|x^p-z^p|}\right|. \qquad (13)
\end{aligned}
$$

That is expression (13), which is the first part of expression (9), if $\left|\sqrt[p]{|sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|}\right| \le 1$, satisfies requirement (4) for addition of contrasts (10) .

Under the condition $\left|\sqrt[p]{|sign\left(C(x,y)\right)|C(x,y)|^p + sign\left(C(y,z)\right)|C(y,z)|^p|}\right| > 1$ the result of contrast addition is always equal to maximal or minimum possible value that is 1 or -1. Thus, in general, expression (8) fulfils requirements of the addition rule (4) and is the method for addition $\breve{+}$ of two contrasts, which are determined by the formula (7). Let us notice that expresion (8) under the conditions $C(x,y) \ge 0, \quad C(y,z) \ge 0$, match with Yager s-norm.

Knowing the addition rule for two contrasts (8) and using the approach described in papers [2, 6] we can find the expression for determination of generalized contrast of two-element image at adaptation of image elements to the gray levels x_0 (x_0 for the case of reduced to the range [0,1] gray levels of the image pixels, or L_0 at direct use of pixels gray levels (brightness) then $L_0 \in [0,\ LMAX]$, where $LMAX = 2^n - 1$, n – number of bits, that represent gray level of pixel). Therefore further we will examine structural approach to the determination of image generalized contrast based on contrast (7).

3 Determination of image generalized contrast

For the determination of image generalized contrast in whole at first we will find the way of calculation of generalized contrast of two-element image on determination of element contrast by expression (7). Knowing it we can constitute the computation way of image generalized contrast choosing for the contrast quantitative estimation of subject image C^Y_{gen} the expectation value of contrast by the analysis of image contrast density distribution

$$
C^Y_{gen} = \int\limits_{-1}^{+1} |C_{ij0}|\cdot p\left(C_{ij0}\right) dC_{ij0}, \qquad (14)
$$

where C_{ij0} is the generalized contrast of two-element image with adaptation to gray level x_0.

For this purpose we will consider two image elements, which can be characterized by the gray levels x_i, y_i and by the value of adaptation to the gray level x_0. Then for those two elements the expressions that outline their contrast with respect to the adaptation level x_0 can be written in such a way:

$$
C_{i0}(x_i,x_0) = sign\left(x_i^p - x_0^p\right)\cdot\left|\sqrt[p]{|x_i^p - x_0^p|}\right|, \quad C_{j0}(x_j,x_0) = sign\left(x_j^p - x_0^p\right)\cdot\left|\sqrt[p]{|x_j^p - x_0^p|}\right|. \qquad (15)
$$

Then percepted by the eye contrast C_{ij0} in the process of viewing such two-element image we can find using the rule (8) of addition of contrasts C_{i0} and C_{j0} (15). Taking it into consideration we will receive:

$$C_{ij0} = \begin{cases} sign\left(sign\left(C_{i0}(x_i,x_0)\right)\left|C_{i0}(x_i,x_0)\right|^p + sign\left(C_{j0}(x_j,x_0)\right)\left|C_{j0}(x_j,x_0)\right|^p\right) \times \\ \times \left|\sqrt[p]{\left|sign\left(C_{i0}(x_i,x_0)\right)\left|C_{i0}(x_i,x_0)\right|^p + sign\left(C_{j0}(x_j,x_0)\right)\left|C_{j0}(x_j,x_0)\right|^p\right|}\right|, \\ \text{if } \left|\sqrt[p]{\left|sign\left(C_{i0}(x_i,x_0)\right)\left|C_{i0}(x_i,x_0)\right|^p + sign\left(C_{j0}(x_j,x_0)\right)\left|C_{j0}(x_j,x_0)\right|^p\right|}\right| \le 1, \\ 1 \text{ , if } \left|\sqrt[p]{\left|sign\left(C_{i0}(x_i,x_0)\right)\left|C_{i0}(x_i,x_0)\right|^p + sign\left(C_{j0}(x_j,x_0)\right)\left|C_{j0}(x_j,x_0)\right|^p\right|}\right| > 1, \end{cases} \tag{16}$$

or

$$\begin{aligned} C_{ij0} &= 0.5 \cdot sign\left(sign\left(C_{i0}(x_i,x_0)\right)\left|C_{i0}(x_i,x_0)\right|^p + sign\left(C_{j0}(x_j,x_0)\right)\left|C_{j0}(x_j,x_0)\right|^p\right) \times \\ &\times \left[\left|\sqrt[p]{\left|sign\left(C_{i0}(x_i,x_0)\right)\left|C_{i0}(x_i,x_0)\right|^p + sign\left(C_{j0}(x_j,x_0)\right)\left|C_{j0}(x_j,x_0)\right|^p\right|}\right| + 1 - \right. \\ &- \left.\left|\left|\sqrt[p]{\left|sign\left(C_{i0}(x_i,x_0)\right)\left|C_{i0}(x_i,x_0)\right|^p + sign\left(C_{j0}(x_j,x_0)\right)\left|C_{j0}(x_j,x_0)\right|^p\right|}\right| - 1\right|\right], \end{aligned} \tag{17}$$

where from

$$\begin{aligned} C_{ij0} &= 0.5 \cdot sign\left(sign\left(x_i^p - x_0^p\right)\left|x_i^p - x_0^p\right| + sign\left(x_j^p - x_0^p\right)\left|x_j^p - x_0^p\right|\right) \times \\ &\times \left[\left|\sqrt[p]{\left|sign\left(x_i^p - x_0^p\right)\left|x_i^p - x_0^p\right| + sign\left(x_j^p - x_0^p\right)\left|x_j^p - x_0^p\right|\right|}\right| + 1 - \right. \\ &- \left.\left|\left|\sqrt[p]{\left|sign\left(x_i^p - x_0^p\right)\left|x_i^p - x_0^p\right| + sign\left(x_j^p - x_0^p\right)\left|x_j^p - x_0^p\right|\right|}\right| - 1\right|\right]. \end{aligned} \tag{18}$$

For the contrast definition of subject image it is necessary to consider all set of pixels pairs x_i and y_i or corresponding contrasts C_{i0} and C_{j0}. Thanks to this the set of values C_{ij0} can be calculated and distribution $p(C_{ij0})$ (histogram of distribution density) that determines the probability that given contrast value C_{ij0} lays in some predefined range of contrasts can be constructed. While analyzing digital images of size $N \times N$ pixels the set of values C_{ij0} creates matrix of N^2 elements.

Calculation of histogram $p(C_{ij0})$ and contrast C_{gen}^Y requires the considerable amount of computations. For the simplification of computations the equal density domains can be extracted on the image and their contrasts can be computed concerning the adaptation level x_0. In such a case all pixels of equal gray levels should be gathered together and it can be postulated that $x_i = x_j = x$; $C_{i0} = C_{j0}$; $C_{ij0} = C_{ji0}$. At that we will have

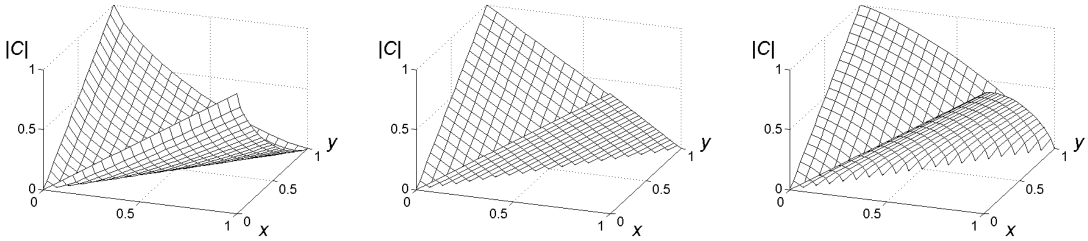
$$C_{ij0} = \frac{1}{2} sign\left(x^p - x_0^p\right)\left[\left|\sqrt[p]{2\left|(x^p - x_0^p)\right|}\right| + 1 - \left|\left|\sqrt[p]{2\left|(x^p - x_0^p)\right|}\right| - 1\right|\right]. \tag{19}$$

And for the calculation of subject image contrast only distribution (histogram $p(x_i) = p(x_j) = p(x)$, where $0 < x < 1$) will be used. This contrast we will take for the image generalized contrast C_{gen}^p, and it will be determined by the expression

$$C_{gen}^p = \frac{1}{2}\int_0^{XMAX}\left|\left|\sqrt[p]{2\left|(x^p - x_0^p)\right|}\right| + 1 - \left|\left|\sqrt[p]{2\left|(x^p - x_0^p)\right|}\right| - 1\right|\right| p(x)\, dx, \tag{20}$$

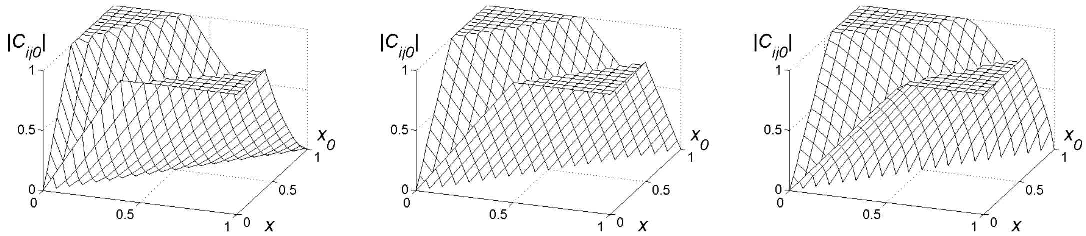
where $XMAX = 1$.

The shapes of contrast modulus function (7) and modulus of two contrasts sum (19) are illustrated on Figures 1 and 2 at different values of parameter p.



(a) p =0.75 (b) p =1 (c) p =1.25

Fig. 1. Graphs of contrast modulus (7) at different values of parameter p.



(a) p =0.75 (b) p =1 (c) p =1.25

Fig. 2. Graphs of modulus of contrasts sum C_{ij0} (19) at different values of parameter p.

4 Experimental investigations of image generalized contrast

For investigation of proposed method effectiveness three images were chosen (Fig. 3 a-3 c) and their generalized contrast C^p_{gen} was examined at different values of parameter p: 0.75, 1.0 and 1.25.

The results of generalized contrast calculation for seven another test images at different values of parameter p are given in the table.

Tab. 1. Generalized contrast C^p_{gen} of test images for different values of parameter p.

Image	***p*=0.75**	***p*=1.00**	***p*=1.25**
tire.tif	0.36	0.38	0.39
bacteria.tif	0.14	0.21	0.28
cameraman.tif	0.33	0.41	0.47
eight.tif	0.21	0.34	0.46
pout.tif	0.08	0.16	0.23
rice.tif	0.17	0.27	0.34
shot1.tif	0.19	0.30	0.40

All those images also definitely confirm the logical tendency of generalized contrast value change from the smallest when p =0.75 to the largest when p =1.25. At that values of contrast received at p =1 are greater then received at p =0.75 and less then received at p =1.25. That is the monotony of general contrast increase at the increment of value p is confirmed. The validity of proposed approach to image generalized contrast determination on the base of contrast (7) is additionaly verified by this.

Proposed approach confirmed interrelation of contrast determination method with fundamental bases of fuzzy logic, namely with triangular s-norms (t-conorms). The fact that expression based on Yager

(a) $C_{gen}^{0.75} = 0.26$, $C_{gen}^{1} = 0.34$, $C_{gen}^{1.25} = 0.40$ (b) $C_{gen}^{0.75} = 0.19$, $C_{gen}^{1} = 0.30$, $C_{gen}^{1.25} = 0.38$ (c) $C_{gen}^{0.75} = 0.11$, $C_{gen}^{1} = 0.23$, $C_{gen}^{1.25} = 0.34$

Fig. 3. Test images and their contrast values at power description of its element contrast (7) for different values of parameter p (0.75, 1.0, 1.25).

s-norm [7] is the rule for addition of contrast of appearance (7) opens new possibilities for application of fuzzy logic at quantitative evaluation of image quality.

5 Conclusion

The interrelation between fuzzy logic, acting as triangular s-norm, and the determination of quantitative evaluation of image generalized contrast is established. It allows to develop methods for evaluation of image perception by sensors with different sensitivity characteristics and to construct new methods for correction of those characteristics and create new algorithms for image quality enhancement based on local contrast amplification . In whole such direction of investigations contributes to more effective realization of intelligent automatic systems for image analysis.

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